Kuwait University
Final Exam
Math 261 - Abstract Algebra

Mathematics Department
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Calculators and communication devices are not allowed. Give full reasons for your answer.

1. (4 pts.) Let G be a non-abelian group of order 25. Show that $a^5 = e$ for each $a \in G$.

2. (4 pts.) Let G and H be two isomorphic groups. Show that if G is cyclic, then H is cyclic as well.

3. (4 pts.) Prove that if M and N are normal subgroups of a group G, then $M \cap N$ is a normal subgroup of G.

4. (4 pts.) Let $\theta : G \to H$ be a group homomorphism from G onto H with $\ker(\theta) = K$. Show that the mapping $\Phi : G/K \to H$ defined by $\Phi(aK) = \theta(a)$ for each $aK \in G/K$ is an isomorphism.

5. (3+3 pts.)
   (a) Simplify: $\big([2],(1\ \ 2\ \ 3)\big)^{-1}\big([1],(2\ \ 4)\big)\big([2],(1\ \ 2\ \ 3)\big)$ in $\mathbb{Z}_4 \times S_4$.
   (b) Find $x \in S_6$ so that: $(1\ \ 3\ \ 6)^{-1}\ x\ (1\ \ 3\ \ 4\ \ 5)^{-1} = (1)$.

6. (3+3 pts.) Let $G = \{2^m \ : \ m \in \mathbb{Z}\}$.
   (a) Show that G is a group with respect to multiplication..
   (b) Show that $\mathbb{Z}$ is isomorphic to G.

7. (3+3 pts.) Let H be a subgroup of a group G and define a relation $\sim$ on G by $a \sim b$ iff $a^{-1}b \in H$.
   (a) Show that $\sim$ is an equivalence relation on G.
   (b) Describe the equivalence classes of $\sim$ on G and if G is finite, determine the number of such classes.

8. (3+3 pts.) Let $GL_n(\mathbb{R})$ denote the set of all $n \times n$ non-singular matrices with real entries, and $SL_n(\mathbb{R}) = \{A \in GL_n(\mathbb{R}) \mid \det(A) = 1\}$.
   (a) Show that $\theta : GL_n(\mathbb{R}) \to \mathbb{R}^*$ defined by $\theta(A) = \det(A)$ for each $A \in GL_n(\mathbb{R})$ is a homomorphism onto R^*.
   (b) Show that $GL_n(\mathbb{R})/SL_n(\mathbb{R})$ is isomorphic to $\mathbb{R}^*$.