Kuwait University
Final Exam
Math 261 - Abstract Algebra

Mathematics Department
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Calculators and communication devices are not allowed. Give full reasons for your answer.

1. (4 pts.) Choose **only** part (a) or (b):

   (a) Let $S = \{1, 2, 3, 4, 5\}$, $G = S_5$, and $T = \{5\}$. Find G_T, the element-wise stabilizer of T in G.

   (b) Let G be a non-abelian group of order 49. Show that $a^7 = e$ for each $a \in G$.

2. (4 pts.) Show that group isomorphism is a symmetric relation.

3. (4 pts.) Show that every subgroup of index 2 is normal.

4. (4 pts.) Let a be an element of a group G. Show that $C(a)$, the centralizer of a in G, is a subgroup of G. Find $C(e)$.

5. (3 + 3 pts.) For an integer $n > 1$, define $\theta : \mathbb{Z} \to \mathbb{Z}_n$ by $\theta(a) = [a]$ for each $a \in \mathbb{Z}$.

   (a) Find $\ker \theta$.

   (b) Is $\mathbb{Z}/ < n >$ isomorphic to $\mathbb{Z}_n$? Explain your answer.

6. (2 pts. each) Let n be an integer so that $n > 1$.

   (a) Show that $\mathbb{U}_n$ is closed under the operation $\odot$.

   (b) Use the Euclidean algorithm to compute the greatest common divisor of 13 and 40, and write it as a linear combination of 13 and 40.

   (c) Find the order of $\mathbb{U}_{95}$.

7. (3+3 pts.) Let $\theta : G \to H$ be a group homomorphism.

   (a) Show that θ is one-to-one if $\ker \theta = \{e_G\}$.

   (b) Show that $\ker \theta$ is normal subgroup of G.

8. (6 pts.) Let M and N be normal subgroups of a group G and let $\gcd(|M|, |N|) = 1$. Show that $mn = nm$ for all $m \in M$ and for all $n \in N$.