Kuwait University
Faculty of Science
Department of Mathematics

Euclidean Geometry

0410-226
First Exam

Monday, October 21, 2019
Fall 2019/2020

Name										
ID Number										
Serial Number										

<u>Duration</u> 75 minutes (This exam contains **5** questions).

Section No.	Instructor Name
1	**Dr. Abdullah Alazemi**

Calculators and communication devices are not allowed in the examination room.
Give full reasons for your answer and State clearly any Theorem you use.

Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Total	

1. (**3 pts. each**) Let $\overline{AC}$ be an angle bisector for the angles $B\hat{A}D$ and $B\hat{C}D$.

(a) Show that $\triangle\, ABC$ is congruent to $\triangle\, ADC$.

(b) Show that $\overline{AC}$ is perpendicular to $\overline{BD}$.

<u>**Solution**</u>:

(a) Clearly, in triangles $\triangle\, ABC$ and $\triangle\, ADC$ we have:

i. $\hat{1} \cong \hat{2}$ (given).

ii. $\hat{5} \cong \hat{6}$ (given).

iii. $\overline{AC}$ is common.

Then, by ASA we have $\triangle\, ABC \cong \triangle\, ADC$.

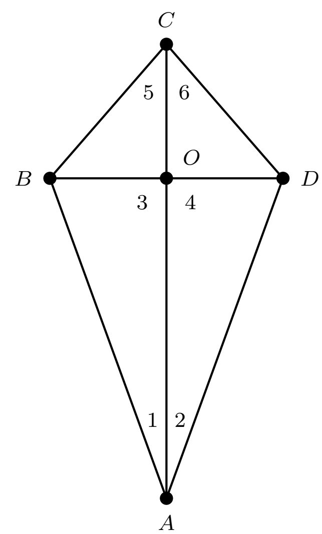
(b) In part (a), we proved that $\overline{AB} \cong \overline{AD}$. Hence, in triangles $\triangle\, ABO$ and $\triangle\, ADO$ we have:

i. $\overline{AB} \cong \overline{AD}$ - proved in part(a).

ii. $\overline{AO}$ is common segment.

iii. $\hat{1} \cong \hat{2}$ (given).

By SAS, we have $\triangle\, ABO \cong \triangle\, ADO$. Therefore, $A\hat{O}B \cong A\hat{O}D$ and hence both are right angles which shows that $\overline{AC} \perp \overline{BD}$.



2. (**3 pts. each**) Let $\triangle\, ABC$ be a triangle with M is the midpoint for the segment $\overline{AB}$. Show that

(a) If $\overline{BC} \parallel \overline{MN}$ then N is the midpoint for $\overline{AC}$.

(b) If N is the midpoint for $\overline{AC}$, then $\overline{BC} \parallel \overline{MN}$.

<u>**Solution**</u>:

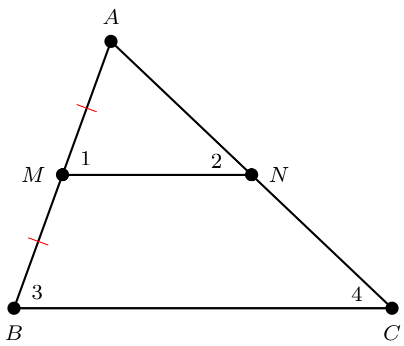
(a) Suppose $\overline{BC} \parallel \overline{MN}$. We show that $\triangle\, ABC \sim \triangle\, AMN$.

i. $\hat{1} \cong \hat{3}$ and $\hat{2} \cong \hat{4}$ (corresponding angles).

ii. $\hat{A}$ is common.

Thus, by S-AA, we have $\triangle\, ABC \sim \triangle\, AMN$.

Hence, $\dfrac{|\overline{AN}|}{|\overline{AC}|} = \dfrac{|\overline{AM}|}{|\overline{AB}|} = \frac{1}{2}$. Therefore, $|\overline{AN}| = \frac{1}{2}|\overline{AC}|$ and therefore, N is the midpoint of $\overline{AC}$.



(b) Suppose N is the midpoint of $\overline{AC}$. In triangles $\triangle\, ABC$ and $\triangle\, AMN$ we have:

i. $\hat{A}$ is common angle.

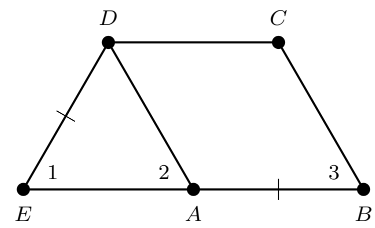
ii. $\dfrac{|\overline{AM}|}{|\overline{AB}|} = \dfrac{|\overline{AN}|}{|\overline{AC}|} = \frac{1}{2}$.

Thus, by S-SAS, we have $\triangle\, ABC \sim \triangle\, AMN$.

Therefore, $\hat{1} \cong \hat{3}$ (corresponding angles) which implies that $\overline{BC} \parallel \overline{MN}$.

3. (**4 pts.**) Let $ABCD$ be a parallelogram. Show that if $\overline{AB} \cong \overline{DE}$, and $\hat{1} \cong \hat{2}$, then $ABCD$ is a rhombus.

<u>Solution</u>:



Assume that $\overline{AB} \cong \overline{DE}$. Then, observe that $\triangle\, DAE$ is isosceles since its base angles $\hat{1}$ and $\hat{2}$ are congruent (given). Thus, $\overline{AD} \cong \overline{DE}$. That is $\overline{AB} \cong \overline{AD}$ and thus $ABCD$ is rhombus (all sides are congruent).

4. **(3+2+1+3 pts.)** In the diagram, let $\overline{EF}$ be a tangent segment to the circle $\odot O$ at the tangency point B. Let $\overline{PA}$ and $\overline{PB}$ be two secants passing through the points D and C, respectively. Assume that the two chords $\overline{AD}$ and $\overline{BC}$ are congruents. Let $(D\hat{P}C)$ $\left|\hat{P}\right| = 15°$. Moreover, let $\left|\widehat{\text{AB}}\right| = 60°$, $\left|\widehat{\text{CD}}\right| = x$, $\left|D\hat{O}C\right| = y$, $\left|\widehat{\text{AD}}\right| = z$ and $\left|\widehat{\text{BC}}\right| = w$.

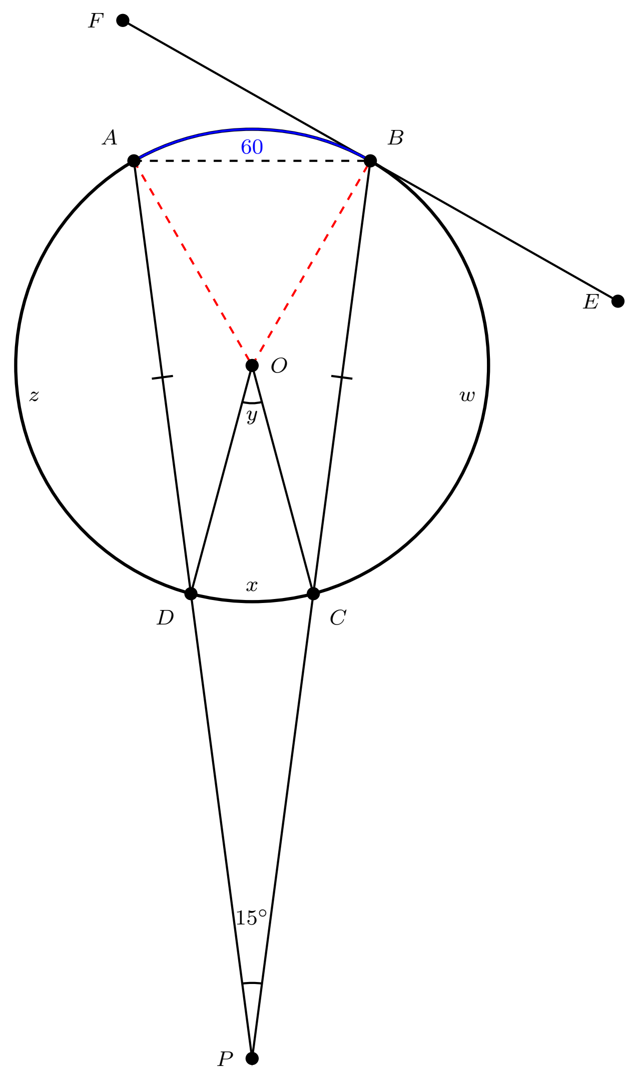
(a) Find the values for x, y, z and w.

(b) Show that $\triangle\ ADO \cong \triangle\ BCO$.

(c) Find $\left|F\hat{B}A\right|$.

(d) Show that $\triangle\ ABO$ is an equiangular triangle.

Solution:



(a) Clearly, $x = y$ (central angle) and $z = w$ (since $\widehat{\text{AD}} \cong \widehat{\text{BC}}$). Note that $\left|\hat{P}\right| = 15° = \frac{1}{2}(60 - x)$ (Theorem in the class) which implies that $60° - x = 30°$ and hence $x = 30°$. Therefore, $y = 30°$.
Moreover, $60° + x + z + w = 60° + 30° + 2z = 360°$ implies that $z = \frac{1}{2}(\,360 - 90\,) = 135°$. Therefore, $z = w = 135°$.

(b) In the triangles $\triangle\ ADO$ and $\triangle\ BCO$, we have:

i. $\overline{AO} \cong \overline{BO}$ (both are radii).

ii. $\overline{DO} \cong \overline{CO}$ (both are radii).

iii. $\overline{AD} \cong \overline{BC}$ (since $\widehat{\text{AD}} \cong \widehat{\text{BC}}$).

By SSS, $\triangle\ ADO \cong \triangle\ BCO$.

(c) Note that the angle $F\hat{B}A$ is an angle between a tangent line and a chord. Therefore, $\left|F\hat{B}A\right| = \frac{1}{2}\left|\widehat{\text{AB}}\right| = \frac{1}{2}60° = 30°$.

(d) Note that $\left|F\hat{B}A\right| = 30°$. Hence $\left|A\hat{B}O\right| = 60°$ (since $F\hat{B}O$ is a right angle). But the triangle $\triangle\ ABO$ is isosceles. Therefore, $\left|B\hat{A}O\right| = 60°$. Moreover, $\left|A\hat{O}B\right| = 60°$. Hence, $\triangle\ ABO$ is an equiangular triangle.

5. (**3 pts.**) **Bonus Question**: Find the "Secret Word".

Cross the following words to find the secret word!?

MATH - BOOK - IN - EUCLID - BEST - CLASS

S	**M**	**A**	**T**	**H**	**I**
B	**O**	**O**	**K**	**A**	**N**
E	**U**	**C**	**L**	**I**	**D**
B	**B**	**E**	**S**	**T**	**A**
C	**L**	**A**	**S**	**S**	**H**

The "Secret Word" is ....