Kuwait University
Faculty of Science
Department of Mathematics

Math 250
Foundations of Mathematics
Spring 2022/2023

Final Exam
Monday, May 15, 2023

Name										
ID Number										

Duration 2 hours (This exam contains **6** questions).

Section No.	Instructor Name
1	**Dr. Abdullah Alazemi**

Give full reasons for your answer and State clearly any Theorem you use.

Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Question 6	
Total	40

1. (**4 pts.**) Let $A = (3, 4) \cup [5, 6)$. Without using the horizontal line test, show that $A \approx (0, 1)$ and find its cardinality.

2. (**4 pts.**) Let $a_1 = 1$, $a_2 = 1$ and $a_{n+2} = a_{n+1} + a_n$ for all $n \in \mathbb{N}$. Show that a_{3n+1} is an odd number for all natural number n.

3. (**4 pts.**) Show that A is a countable set, where

$$A = \left\{ \frac{1}{2k+3} \, : \, k \in \mathbb{N} \right\}.$$

4. (**8 pts.**)

(a) Let $f : A \to \mathbb{N}$ be a function defined by $f((\, m, n\,)) = m$, where $A = \{\, (\, m, n\,) \in \mathbb{N} \times \mathbb{R} : n = m\pi \,\}$. Show that f is a bijection.

(b) Let $g : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be a function given by $g(a, b) = a + b$ for all $a, b \in \mathbb{N}$. Decide whether g is one-to-one and onto $\mathbb{N}$.

5. (**8 pts.**)

(a) Let A, B and C be three nonempty sets. Let $f : A \to B$, $g : B \to C$ and $h : B \to C$ be any three functions with $g \circ f = h \circ f$. Show that if f is **onto** B, then $g = h$.

(b) Let A be a set so that $f : \mathbb{N} \to A$ is a bijection. For any element $x \notin A$, use the one-to-one function $g : \mathbb{N} \to A \cup \{x\}$ defined by

$$g(n) = \begin{cases} x & \text{if } n = 1 \\ f(n-1) & \text{if } n > 1 \end{cases},$$

to show that $A \cup \{x\}$ is countable.

6. (**10 pts.**) Let $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be a bijection defined by $f((\, m, n\,)) = 2^{m-1}(\, 2n - 1\,)$.

(a) Show that if $A \approx C$ and $B \approx D$, then $A \times B \approx C \times D$.

(b) Show that if A and B are two denumerable sets, then $A \times B$ is denumerable as well.

(c) Find the inverse image of $Y = \{\, 5, 8\,\}$.

- (**2 pts.**) **Bonus Question:**

  Let $\mathcal{R}$ be some relation on a nonempty set A, and let $\mathcal{S}$ be a transitive relation containing $\mathcal{R}$. Show that $\mathcal{R} \circ \mathcal{R} \subseteq \mathcal{S}$.