1. (**4 pts.**) Let P and Q be two propositional forms. Show that $P \Rightarrow Q$ is equivalent to its contrapositive, but it is not equivalent to its converse.

Solution:

2. (**2.5+2.5 pts.**)

(a) Let $P(x)$ be an open sentence with a variable $x \in \mathcal{U}$ for some universe $\mathcal{U}$. Show that the nagation of "$(\exists\, x)[P(x)]$" is equivalent to "$(\forall\, x)[\sim P(x)]$"

(b) Find a denial for "$(\forall\, x)(\exists\, y)\Big[\big((x \le y-1) \;\vee\; (x=y)\big) \wedge (x>1)\Big]$"

Solution:

3. (**2+3+3 pts.**)

(a) Let $a, b \in \mathbb{Z}$. Use a contrapositive proof to show that if ab-odd, then a-odd and b-odd.

(b) Show that the set of primes in $\mathbb{N}$ is infinite.

(c) Let $A = \{ 9^n : n \in \mathbb{Z} \}$ and $B = \{ 3^n : n \in \mathbb{Z} \}$. Show that $A \subsetneq B$.

Solution:

4. (**2+3+3 pts.**)

(a) Let $A = \{1, \{2\}, 3\}$. Compute the power set of A, $\mathcal{P}(A)$.

(b) Show that if A and B are two sets, then $\widetilde{A \cap B} = \widetilde{A} \cup \widetilde{B}$.

(c) Suppose that A and B are two nonempty sets. Show that if $A - B = \phi$ then $A \cap B = A$.

<u>**Solution**</u>: