



جامعة الكويت
KUWAIT UNIVERSITY

Kuwait University
Faculty of Science
Department of Mathematics

Math 250

Foundations of Mathematics

Spring 2022/2023

First Exam
Monday, Mar 20, 2023

Name										
ID Number										

Duration 75 minutes (This exam contains 5 questions).

Section No.	Instructor Name
1	Dr. Abdullah Alazemi

Give full reasons for your answer and State clearly any Theorem you use.

Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Total	50

1. (8 pts.)

(a) Let $a, b, c \in \mathbb{Z}$. Show that if $a \mid b$ and $a \mid c$, then $a \mid (b - c)$.

(b) Let W be the set of all prime numbers in \mathbb{N} . Show that W is infinite.

2. (8 pts.) Use a truth table to show that $(\mathbf{P} \Rightarrow \mathbf{Q}) \Leftrightarrow ((\sim \mathbf{Q}) \Rightarrow (\sim \mathbf{P}))$ is a tautology, for any propositions \mathbf{P} and \mathbf{Q} .

3. (10 pts.) For each $i \in \mathbb{N}$, define $A_i = \{2i + 1 \in \mathbb{N}\}$ and $B_i = \{j \in \mathbb{N} : j = 1 \text{ or } j > i\}$.

(a) Find A_1, A_2, A_3 and $\bigcap_{i \in \mathbb{N}} \widetilde{A}_i$.

(b) Find B_1, B_2, B_3 and $\bigcup_{i \in \mathbb{N}} \widetilde{B}_i$.

4. (12 pts.) Let A and B be two nonempty sets.

(a) Show that $A - B = A \cap \tilde{B}$.

(b) Let $X = \{9^n : n \in \mathbb{Z}\}$ and $Y = \{3^n : n \in \mathbb{Z}\}$. Show that $X \subsetneq Y$.

5. (12 pts.)

(a) Show that for all $n \in \mathbb{N}$, $3 \mid n^3 + 5n$.

(b) Show that for all natural number $n \geq 7$, $n! > 3^n$.

