Kuwait University
Faculty of Science
Department of Mathematics

Math 250
Foundations of Mathematics
Spring 2022/2023

Second Exam
Monday, May 01, 2023

Name										
ID Number										

Duration 75 minutes (This exam contains **5** questions).

Section No.	Instructor Name
1	**Dr. Abdullah Alazemi**

Give full reasons for your answer and State clearly any Theorem you use.

Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Total	/50

1. (**10 pts.**) Let $\mathcal{R}$ be a relation on a nonempty set A. Show that $\mathcal{R} \cup \mathcal{R}^{-1}$ is a symmetric relation.

2. (**10 pts.**) Let $f : A \to B$ and $g : B \to C$ be two functions.

(a) Show that if $(x, y), (x, z) \in g \circ f$, then $y = z$. **"Do not assume that $g \circ f$ is a function!!"**

(b) Show that if f^{-1} is a function, then f^{-1} is one-to-one.

3. (**10 pts.**) Prove or Disprove the following statements:

(a) For any two sets A and B, if $A \times B = \phi$, then A or B is the emptyset.

(b) If $\mathcal{R}$ is the relation on $\mathbb{Z}$ given by $m\mathcal{R}n \Leftrightarrow m$ divides n, then $\mathcal{R}$ is an antisymmetric relation.

4. (**10 pts.**) Let f be a relation from $M_{2\times 2}$, the set of all 2×2 matrices whose entries are real numbers, to $\mathbb{R}$ defined by $f(A) = |A|$.

(a) Decide whether $f : M_{2\times 2} \to \mathbb{R}$ is a function.

(b) If f is a function, then decide whether f is one-to-one and onto $\mathbb{R}$.

5. (**10 pts.**) Let $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be an onto function defined by $f\big((m, n)\big) = 2^{m-1}(2n - 1)$. Show that f is a bijection.