

Kuwait University Faculty of Science Department of Mathematics

Abstract Algebra I 0410-261 Final (On-Line) Exam

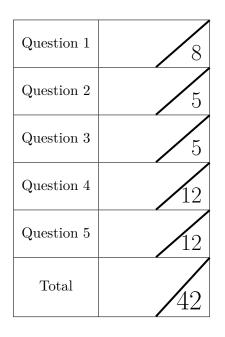
Monday, September 28, 2020 Spring 2019/2020

Name					
ID Number					

Duration 2 hours [11:00 - 13:00] (This exam contains 5 questions).

Section No.	Instructor Name				
1	Dr. Abdullah Alazemi				

Give full reasons for your answer and State clearly any Theorem you use. You are not allowed to open books/notes or any similar resources during the exam.



Good Luck

- 1. (2+3+3 pts.) Let G be a group of a finite order.
 - (a) Show that G has a unique identity.
 - (b) Show that if H and K are two subgroups of G, then $|H \cap K|$ divides |G|.
 - (c) Show that if G is a non-abelian group of order 10, then G has an element of order 5.
- 2. (2+3 pts.)
 - (a) List the isomorphism class representatives of abelian groups of order 100.
 - (b) Let G and H be two gourps. Show that if $\theta : G \to H$ is a group homomorphism with ker $(\theta) = \{e_G\}$, then θ is one-to-one.
- **3.** (2+3 pts.) Let H be a subgroup of a finite group G with [G:H] = 2
 - (a) Show that H is a normal subgroup of G.
 - (b) Compute the order of the quotient group G/H.
- 4. (3 pts. each) Let G and H be two isomorphic groups.
 - (a) Show that if $\theta: G \to H$ is an isomorphism then $\theta(G) \leq H$.
 - (b) Show that if G is abelian, then H is abelian as well.
 - (c) Show that if $f : G \to H$ is an isomorphism and B is a normal subgroup of H, then the subgroup $A = f^{-1}(B)$ of G is also normal in G.
 - (d) Show that if $|G| = p^2$ (p is prime number), then G has at least one subgroup of order p.
- 5. (3 pts. each) Let $GL_n(\mathbb{R}) = \{ \text{ all } n \times n \text{ nonsingular matrices with real entries} \}$ be a group with the operation of matrix multiplication.
 - (a) Show that $SL_n(\mathbb{R}) = \{ A \in GL_n(\mathbb{R}) : |A| = 1 \}$ is a subgroup of $GL_n(\mathbb{R})$.
 - (b) Find the centralizer of $X = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ in $GL_2(\mathbb{R})$, denoted as C(X).
 - (c) Show that $\theta: GL_n(\mathbb{R}) \to \mathbb{R}^*$ defined by $\theta(A) = |A|$ for each $A \in GL_n(\mathbb{R})$ is a homomorphism onto \mathbb{R}^* .
 - (d) Use the Fundamental Homomorphism Theorem to show that $GL_n(\mathbb{R})/SL_n(\mathbb{R})$ is isomorphic to \mathbb{R}^* .