



Kuwait University
Faculty of Science
Department of Mathematics

Abstract Algebra I

0410-261

First On-Line Midterm

Monday, August 17, 2020
Spring 2019/2020

Name										
ID Number										

Duration 75 minutes (This exam contains 4 questions).

Section No.	Instructor Name
1	Dr. Abdullah Alazemi

Give full reasons for your answer and State clearly any Theorem you use.

Question 1	7
Question 2	5
Question 3	4
Question 4	5
Total	21

Good Luck

1. (3+2+2 pts.) Let $n\mathbb{Z} = \{na : a \in \mathbb{Z}\}$ for any $n \in \mathbb{Z}$.
- (a) Show that $(5\mathbb{Z}, +)$ is an abelian group.
 - (b) Is $(2\mathbb{Z} \cup 3\mathbb{Z}, +)$ a group? Explain.
 - (c) Let $G = \{A \in M_{2 \times 2} : \det A = 2\}$. Is G a group under the matrix multiplication? Explain.
2. (1+2+2 pts.) In S_7 :
- (a) Decide whether $\alpha = (1 \ 3 \ 5)(2 \ 7 \ 4)$ is an even or an odd permutation.
 - (b) Find the cyclic decomposition of $\beta = (1 \ 3 \ 7)(2 \ 3 \ 5 \ 7)(4 \ 5 \ 6)$.
 - (c) Compute $\gamma = (1 \ 3 \ 7)(2 \ 6 \ 5 \ 4)^{-1}(1 \ 3 \ 7)^{-1}$.
3. (3+1 pts.) Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $G = S_{10}$, and $T = \{1, 7\} \subseteq S$.
- (a) Find $G_{(T)}$, which leaves T setwise invariant, and find its order.
 - (b) Show that $G_{(T)}$ is not contained in G_T , that is $G_{(T)} \not\subseteq G_T$.
4. (2+1+2 pts.) Let G be a group.
- (a) Let $H, K \leq G$ and $HK = \{hk : h \in H \text{ and } k \in K\}$. Is HK a subgroup of G ? Explain.
 - (b) If G is abelian, compute $Z(G)$, the center of G .
 - (c) If $(ab)^2 = a^2b^2$, for all $a, b \in G$, then show that G must be abelian.