Kuwait University
Faculty of Science
Department of Mathematics

Math 261
Abstract Algebra I
Summer 2022/2023

First Exam

June 15, 2023

Name										
ID Number										

Duration **60** minutes (This exam contains **4** questions).

Section No.	Instructor Name
1	**Dr. Abdullah Alazemi**

Give full reasons for your answer and State clearly any Theorem you use.

Question 1	
Question 2	
Question 3	
Question 4	
Total	40

1. (**10 pts.**) Which of the following equations define **operations** on the set of integers? Of those that do, which are **associative**? Which are **commutative**? Which have identity element?

(a) $a * b = \dfrac{a+b}{2}$.

(b) $a * b = ab + 1$.

(c) $a * b = a$.

2. (**10 pts.**) Consider S_6, the symmetric group on $\{1, 2, \ldots, 6\}$, in what follows.

(a) Solve for x in

$$(1\ \ 6\ \ 4)\,(2\ \ 3\ \ 5)\,(1\ \ 6\ \ 4) = (2\ \ 3\ \ 5)\,x.$$

(b) Find the cyclic decomposition of $(1\ \ 2\ \ 3)^{-1}\,(4\ \ 5\ \ 6)\,(1\ \ 3\ \ 2)$.

(c) Decide whether $\alpha = (1\ \ 3\ \ 5)\,(2\ \ 4\ \ 6)$ is an even or an odd permutation.

3. (10 pts.) Let G be a group with a binary operation $*$ with the identity e.

(a) Show that the inverse of each element in G is unique.

(b) Let H be a nonempty subset of G such that $a, b \in H$ implies $ab^{-1} \in H$. Show that H is a subgroup of H.

4. (**10 pts.**)

(a) Let $G = \{A \in M_{2\times 2} : \det A = 1\}$. Show that G with the operation of matrix multiplication is a group.

(b) Let $H = \{1, -1\}$. Show that H is a subgroup of $(\mathbb{R}^*, \cdot)$ and find its order.