Abstract Algebra I

0410-261
Second On-Line Midterm

Monday, September 7, 2020
Spring 2019/2020

Name										
ID Number										

Duration 75 minutes (This exam contains **4** questions).

Section No.	Instructor Name
1	**Dr. Abdullah Alazemi**

Give full reasons for your answer and State clearly any Theorem you use.

Question 1	4
Question 2	7
Question 3	5
Question 4	5
Total	21

Good Luck

1. **(3+1 pts.)**

   (a) Let $\alpha : A \to B$ (A and B are two nonempty sets) be a mapping and define a relation $\sim$ on A so that for any $x, y \in A$, $x \sim y$ if and only if $\alpha(x) = \alpha(y)$. Show that $\sim$ is an equivalence relation on A.

   (b) Is $(\mathbb{Z}_4^*, \odot)$ a group? Explain.

2. **(3+2+2 pts.)** Let $\mathbb{U}_n = \{\, [k] : 1 \le k < n \text{ and } GCD(k,n) = 1 \,\}$.

   (a) Use the Euclidean algorithm to find the inverse of $[35]$ in $\mathbb{U}_{72}$.

   (b) What is the order of $[35]$ in $\mathbb{U}_{72}$? Explain.

   (c) Find the order of $\mathbb{U}_{72}$.

3. **(3+2 pts.)** Let G be a group.

   (a) Show that if $G = \langle a \rangle$ for any $a \in G$, then $G = \langle a^{-1} \rangle$.

   (b) Let p and q be two prime numbers. Show that if the order of the group G is pq, then every proper subgroup of G is cyclic.

4. **(3+2 pts.)**

   (a) Compute all distinct left cosets of $H = \langle (1\ \ 3\ \ 2) \rangle \times \langle [1] \rangle$ in $S_3 \times Z_2$.

   (b) Use Fermat's Little Theorem to find the least nonnegative integer x so that $3^{2023} \equiv x \pmod{11}$.