Kuwait University
Faculty of Science
Department of Mathematics

Math 261
Abstract Algebra I
Summer 2022/2023

Second Exam
June 26, 2023

Name										
ID Number										

Duration 60 minutes (This exam contains **3** questions).

Section No.	Instructor Name
1	**Dr. Abdullah Alazemi**

Give full reasons for your answer and State clearly any Theorem you use.

Question 1	
Question 2	
Question 3	
Total	/40

1. (**8 pts.**) Let $S = \{1, 2, \ldots, 10\}$, $G = S_{10}$, and $T = \{1, 3, 5, 7, 9\}$.

(a) Find $G_{(T)}$ and its order.

(b) Let H and K be any two subgroups of G. Does HK form a subgroup of G? Explain.

2. (**14 pts.**) Let $\mathcal{U}_n = \{\, [k] \,:\, 1 \leq k < n \text{ and } gcd(k, n) = 1 \,\}$.

(a) Show that $\mathcal{U}_n$ is closed under the operation $\odot$.

(b) Use the Euclidean algorithm to find the inverse of [21] as the least nonnegative integer in $\mathcal{U}_{100}$.

(c) Find the order of $\mathcal{U}_{100}$.

3. (**18 pts.**) Let G be a group. Consider the subgroup $C(x)$, the centralizer of x in G, and the subgroup $Z(G)$, the center of G.

(a) Find $C(e)$, where e is the identity element of G.

(b) Show that if H and K are two subgroups of G, then $H \cap K$ is a subgroup of G.

(c) For any $a, b \in G$, determine if $C(a) \cap C(b)$ is a subgroup of G. Also, show that $Z(G)$ is a subgroup of $C(a) \cap C(b)$.

(d) If $xy \in Z(G)$ for all $x, y \in G$, then determine if G is an abelian group.