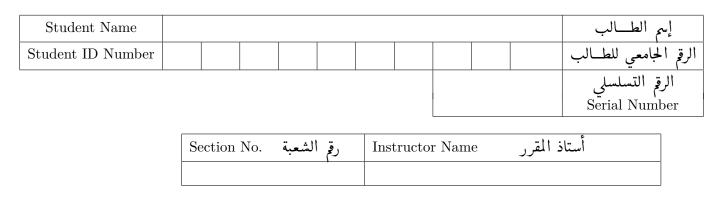


Kuwait University Faculty of Science Department of Mathematics

## Abstract Algebra II

## 0410-262 Second Exam

Monday, November 12, 2018 Fall 2018/19



Instructions to students

Time allowed: 1.25 hours.

This exam contains 4 questions.

ممنوع دخول الألات الحاسبة أو أي وسيلة للإتصال داخل قاعة الإختبار.

Calculators and communication devices are not allowed in the examination room.

Question 1	
Question 2	
Question 3	
Question 4	
Total	

تعليمات للطالب

وقت الإختبار: ساعة وربع. يحتوي هذا الإختبار على 4 أسئلة. **1.** (2+1 pts.) Let f(x) = 2 + 2x and  $g(x) = 2 + 3x - x^2$  in  $\mathbb{Z}_5[x]$ .

- (a) Compute: f(x) g(x) in  $\mathbb{Z}_5[x]$ .
- (b) Compute:  $10(f^2(x) g(x))$  in  $\mathbb{Z}_5[x]$ .
- **2.** (3+2+1 pts. each) In  $\mathbb{Z}_5[x]$ :
  - (a) Divide  $f(x) = 2x^4 + x^2 x + 1$  by g(x) = 2x 1.
  - (b) Use the Remainder Theorem to determine the remainder when  $h(x) = 2x^5 3x^3 + 2x + 1$  is divided by x 2.
  - (c) Is -3 a root for h(x) from part (b)? Explain.

## **3.** (3+2+2 pts.)

- (a) Determine whether  $f(x) = x^3 + x + 1$  is irreducible over  $\mathbb{Z}_3$ . If f(x) is reducible, then express it as a product of irreducible polynomials in  $\mathbb{Z}_3[x]$ .
- (b) Use Eisenstein's Irreducibility Criterion to show that  $f(x) = x^2 + 8x 2$  is irreducible over  $\mathbb{Q}$ .
- (c) Show that for a prime  $p, f(x) = x^p + a \in \mathbb{Z}_p[x]$  is not irreducible for any  $a \in \mathbb{Z}_p$ .

## 4. (3 pts. each)

- (a) Show that if F is a field, then F has no ideals other than (0) and F.
- (b) If R is a commutative ring and  $a \in R$ , show that  $I_a = \{x \in R : ax = 0\}$  is an ideal of R.
- (c) Let  $\theta: R \to S$  be a ring homomorphism. Show that if ker  $\theta = \{0_R\}$ , then  $\theta$  is one-to-one.