



Kuwait University
Faculty of Science
Department of Mathematics

Advanced Linear Algebra

0410-363

First Exam

Monday, March 18, 2019
Spring 2018/19

Student Name									إسم الطالب
Student ID Number									الرقم الجامعي للطالب
									الرقم التسلسلي Serial Number

Section No. رقم الشعبة	Instructor Name أستاذ المقرر
01 A	Dr. Abdullah Alazemi

Instructions to students

تعليمات للطالب

Time allowed: 1.25 hours.

وقت الإختبار: ساعة وربع.

This exam contains 4 questions.

يحتوي هذا الإختبار على 4 أسئلة.

ممنوع دخول الآلات الحاسبة أو أي وسيلة للإتصال داخل قاعة الإختبار.

Calculators and communication devices are not allowed in the examination room.

Question 1	
Question 2	
Question 3	
Question 4	
Total	

1. (3+3 pts.)

(a) Show that the set \mathbb{W} of all symmetric matrices is a subspace of $M_{n \times n}(\mathbb{F})$, for a field \mathbb{F} .

(b) Let $\mathbb{W} = \left\{ \begin{pmatrix} a & a-b \\ a+b & b \end{pmatrix} : a, b \in \mathbb{R} \right\}$.

Show that \mathbb{W} is a subspace of $M_{2 \times 2}(\mathbb{R})$. Find a basis for \mathbb{W} and find its dimension.

2. (3+3 pts.)

(a) Let x and y be distinct vectors of a vector space \mathbb{V} . Show that if $\beta = \{x, y\}$ is a basis for \mathbb{V} and a is a nonzero scalar, then $\gamma = \{x + y, ax\}$ is also a basis for \mathbb{V} .

(b) Let $\beta = \{1 + x, 1 - x, x^2\}$ be an ordered basis for $\mathbb{P}_2(\mathbb{R})$. Find $[3 + 5x + x^2]_{\beta}$.

3. (3+3 pts.)

(a) Let $\mathbb{W} = \{f(x) \in \mathbb{P}_2(\mathbb{R}) : f(0) = f'(0) \text{ and } f(1) = f'(1)\}$. Find a basis for \mathbb{W} .

(b) Let $\mathbb{W} = \{(x, y, x - 2y) : x, y \in \mathbb{R}\}$ be a subset of \mathbb{R}^3 . Show that \mathbb{W} is a subspace for \mathbb{R}^3 .

4. (3+4 pts.)

(a) Show that $\mathbf{T} : \mathbb{R}^2 \rightarrow \mathbb{P}_1(\mathbb{R})$, defined by $\mathbf{T}(a, b) = a + bx$ is linear.

(b) Let $\mathbf{T} : \mathbb{P}_1(\mathbb{R}) \rightarrow \mathbb{P}_2(\mathbb{R})$ be a linear for which $\mathbf{T}(t + 1) = t^2 - t$ and $\mathbf{T}(t - 1) = t^2 + 1$. What is $\mathbf{T}(5t + 1)$? Explain.