Kuwait University
Second Exam
Math 363 - Advanced Linear Algebra

Mathematics Department
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Instructor: A. AlAzemi

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1. (**2+3 pts.**) Let $\mathbf{T}$ be the linear operator on $\mathbb{P}_2(\mathbb{R})$ defined by $\mathbf{T}(f) = f + f(2)\,x$.

(a) Show that the eigenvalues for $\mathbf{T}$ are 1 and 3.

(b) Is $\mathbf{T}$ diagonalizable? Explain your answer.

2. (**2+2+1 pts.**) Let $\mathbf{T}$ be the linear operator on $\mathbb{P}_1(\mathbb{R})$ defined by $\mathbf{T}(a + bx) = 2b + (a+b)\,x$.

(a) Find all eigenvalues for $\mathbf{T}$.

(b) Find all eigenvectors for $\mathbf{T}$.

(c) Find a basis γ for $\mathbf{T}$ so that $[\mathbf{T}]_\gamma$ is a diagonal matrix.

3. (**2+3 pts.**)

(a) Let $\mathbb{V}$ be an inner product space. Show that if $\langle u, v \rangle = \langle u, w \rangle$ for all $u \in \mathbb{V}$, then $v = w$.

(b) Let $\mathbb{V} = \mathbb{C}^3$ with the standard inner product on $\mathbb{V}$. Evaluate $\langle x, y \rangle$ and verify the Cauchy-Schwarz Inequality for $x = (1, i, 1+i)$ and $y = (1-i, 1, 2i)$.

4. (**2+1+2 pts.**) Let $\mathbb{V} = \mathbb{P}_1(\mathbb{R})$ with inner product $\langle f, g \rangle = \int_{-1}^{1} f(t)g(t)\,dt$. Let $\alpha = \{1, 1+x\}$ be a subset in $\mathbb{V}$.

(a) Apply the Gram-Schmidt process on α to construct an orthogonal basis β for $\mathbb{P}_1(\mathbb{R})$.

(b) Normalize the vectors in β to obtain an orthonormal basis γ for $\mathbb{P}_1(\mathbb{R})$.

(c) Compute the Fourier coefficients of $h(x) = a + bx$ relative to γ to write $h(x)$ as a linear combination of the vectors in γ.

5. (**2+2+2 pts.**)

(a) Let $\mathbb{V}$ be an inner product space. Show that if $S = \{x_1, x_2, \cdots, x_n\}$ is an orthogonal subset of $\mathbb{V}$ consisting of nonzero vectors, then S is linearly independent.

(b) Show that if $\mathbb{V}$ is a real inner product space, then $x + y$ is orthogonal to $x - y$ for any $x, y \in \mathbb{V}$ with $\|x\| = \|y\|$.

(c) Decide whether $\langle (a, b), (c, d) \rangle = ac - bd$ is an inner product on $\mathbb{R}^2$.