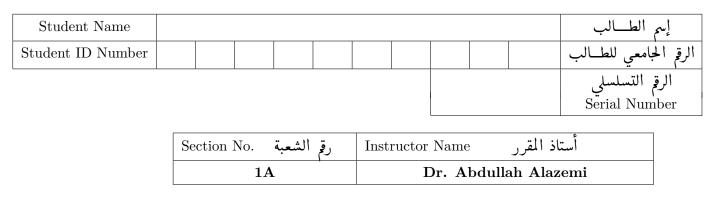


Kuwait University Faculty of Science Department of Mathematics

Advanced Linear Algebra

0410-363 Second Exam

Monday, April 22, 2019 Spring 2018/2019



Instructions to students

Time allowed: 1.25 hours.

This exam contains 4 questions.

ممنوع دخول الألات الحاسبة أو أي وسيلة للإتصال داخل قاعة الإختبار.

Calculators and communication devices are not allowed in the examination room.

Question 1	
Question 2	
Question 3	
Question 4	
Total	

تعليمات للطالب

وقت الإختبار: ساعة وربع. يحتوي هذا الإختبار على 4 أسئلة.

- 1. (5+2 pts.) Let $\mathcal{T}: \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by $\mathcal{T}(x, y, z) = (x, y)$.
 - (1) Find $\mathcal{N}(\mathcal{T})$; $\mathcal{R}(\mathcal{T})$; bases for $\mathcal{N}(\mathcal{T})$, $\mathcal{R}(\mathcal{T})$; $nullity(\mathcal{T})$ and $rank(\mathcal{T})$. Is \mathcal{T} one-to-one? Is \mathcal{T} onto? Explain.
 - (2) Let $\mathbf{U} : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation with $\mathbf{U}((1,1,0)) = (1,1)$, $\mathbf{U}((0,1,1)) = (0,1)$ and $\mathbf{U}((1,0,2)) = (1,0)$. Is \mathbf{U} a one-to-one transformation? Explain.
- 2. (5+2 pts.)
 - (1) Let $\mathcal{T} : \mathbb{P}_1(\mathbb{R}) \to \mathbb{P}_2(\mathbb{R})$ be a linear defined by $\mathbf{T}(f(x)) = x f(x)$. (a): Find a matrix representation A for \mathcal{T} . (b): If h(x) = 3x 2, evaluate $\mathcal{T}(h(x))$ using A.
 - (2) Let $\beta = \{(1,1), (1,-1)\}$ and $\gamma = \{(2,4), (3,1)\}$ be bases for \mathbb{R}^2 . What is the matrix Q that changes γ -coordinates into β -coordinates.
- **3.** (5 pts.) Let \mathcal{T} be the linear operator on $\mathbb{P}_2(\mathbb{R})$ with

$$A = [\mathcal{T}]_{\beta} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix},$$

where $\beta = \{1, x, x^2\}$. If the characteristic polynomial of A is given by $f_A(\lambda) = (\lambda - 1)^2(\lambda - 2)$, find an ordered basis α for $\mathbb{P}_2(\mathbb{R})$ consisting of eigenvectors of \mathcal{T} so that $[\mathcal{T}]_{\alpha}$ is a diagonal matrix.

- 4. (3+3 pts.)
 - (1) Let **U** be a linear operator on a vector space \mathbb{V} over a field **F**, and let λ be an eigenvalue of **U**. Show that the eigenspace E_{λ} is a subspace of \mathbb{V} .
 - (2) Let \mathcal{T} be an operator on $\mathbb{P}_1(\mathbb{R})$ defined by $\mathcal{T}(a+bx) = 2a + (a+b)x$. Compute $A = [\mathcal{T}]_{\beta}$, where $\beta = \{1+x, 1-x\}$. Without computing the eigenvectors, determine whether β is a basis of eigenvectors of \mathcal{T} .