Kuwait University
Faculty of Science
Department of Mathematics

Advanced Linear Algebra

0410-363
Second Exam

Monday, April 22, 2019
Spring 2018/2019

Student Name											إسم الطــالب
Student ID Number											الرقم الجامعي للطــالب
											الرقم التسلسلي Serial Number

Section No. رقم الشعبة	Instructor Name أستاذ المقرر
1A	**Dr. Abdullah Alazemi**

Instructions to students — تعليمات للطالب

Time allowed: 1.25 hours. — وقت الإختبار: ساعة وربع.

This exam contains 4 questions. — يحتوي هذا الإختبار على 4 أسئلة.

ممنوع دخول الآلات الحاسبة أو أي وسيلة للإتصال داخل قاعة الإختبار.
Calculators and communication devices are not allowed in the examination room.

Question 1	
Question 2	
Question 3	
Question 4	
Total	

1. (**5+2 pts.**) Let $\mathcal{T} : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by $\mathcal{T}(x, y, z) = (x, y)$.

(1) Find $\mathcal{N}(\mathcal{T})$; $\mathcal{R}(\mathcal{T})$; bases for $\mathcal{N}(\mathcal{T})$, $\mathcal{R}(\mathcal{T})$; $nullity(\mathcal{T})$ and $rank(\mathcal{T})$. Is $\mathcal{T}$ one-to-one? Is $\mathcal{T}$ onto? Explain.

(2) Let $\mathbf{U} : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation with $\mathbf{U}((1, 1, 0)) = (1, 1)$, $\mathbf{U}((0, 1, 1)) = (0, 1)$ and $\mathbf{U}((1, 0, 2)) = (1, 0)$. Is $\mathbf{U}$ a one-to-one transformation? Explain.

2. (**5+2 pts.**)

(1) Let $\mathcal{T} : \mathbb{P}_1(\mathbb{R}) \to \mathbb{P}_2(\mathbb{R})$ be a linear defined by $\mathbf{T}(f(x)) = x\, f(x)$. ⓐ: Find a matrix representation A for $\mathcal{T}$. ⓑ: If $h(x) = 3x - 2$, evaluate $\mathcal{T}(h(x))$ using A.

(2) Let $\beta = \{ (1, 1), (1, -1) \}$ and $\gamma = \{ (2, 4), (3, 1) \}$ be bases for $\mathbb{R}^2$. What is the matrix Q that changes γ-coordinates into β-coordinates.

3. (**5 pts.**) Let $\mathcal{T}$ be the linear operator on $\mathbb{P}_2(\mathbb{R})$ with

$$A = [\mathcal{T}]_\beta = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix},$$

where $\beta = \{ 1, x, x^2 \}$. If the characteristic polynomial of A is given by $f_A(\lambda) = (\lambda - 1)^2(\lambda - 2)$, find an ordered basis α for $\mathbb{P}_2(\mathbb{R})$ consisting of eigenvectors of $\mathcal{T}$ so that $[\mathcal{T}]_\alpha$ is a diagonal matrix.

4. (**3+3 pts.**)

(1) Let $\mathbf{U}$ be a linear operator on a vector space $\mathbb{V}$ over a field $\mathbf{F}$, and let λ be an eigenvalue of $\mathbf{U}$. Show that the eigenspace E_λ is a subspace of $\mathbb{V}$.

(2) Let $\mathcal{T}$ be an operator on $\mathbb{P}_1(\mathbb{R})$ defined by $\mathcal{T}(a + bx) = 2a + (a + b)x$. Compute $A = [\mathcal{T}]_\beta$, where $\beta = \{ 1 + x, 1 - x \}$. Without computing the eigenvectors, determine whether β is a basis of eigenvectors of $\mathcal{T}$.