Kuwait University
Faculty of Science
Department of Mathematics

Math 403
Advanced Linear Algebra
Spring 2021/2022

First Exam
Wednesday, Apr 25, 2022

Name										
ID Number										

Duration 75 minutes (This exam contains **4** questions).

Section No.	Instructor Name
1	**Dr. Abdullah Alazemi**

Give full justification for your answers and state clearly any theorems you use. Calculators and communication devices are not allowed in the examination room.

Question 1	
Question 2	
Question 3	
Question 4	
Total	50

1. (**5+4+3 pts.**) Let $\mathbf{T} : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $\mathbf{T}(x, y) = (x, x - y)$.

(a) Show that $\mathbf{T}$ is a linear transformation.

(b) Find $\mathcal{N}(\mathbf{T})$ and $\mathcal{R}(\mathbf{T})$.

(c) Is $\mathbf{T}$ invertible? Exaplin.

2. (**5+4+3 pts.**) Let $\mathbf{T} : \mathbb{P}_1(\mathbb{R}) \to \mathbb{P}_2(\mathbb{R})$ be a linear defined by $\mathbf{T}\,(f(x)) = x\,f(x)$.

(a) Find a matrix representation A for $\mathbf{T}$.

(b) Use A, from part (a), to evaluate $\mathbf{T}(\,5x - 1\,)$.

(c) Find a basis for $\mathcal{R}(\mathbf{T})$.

3. (**7+6 pts.**) Let β and γ be the standard ordered bases for $\mathbb{P}_1(\mathbb{R})$ and $\mathbb{R}^2$, respectively.

Let $\mathbf{T} : \mathbb{P}_1(\mathbb{R}) \to \mathbb{R}^2$ and $\mathbf{U} : \mathbb{R}^2 \to \mathbb{P}_1(\mathbb{R})$ be two linear transformations defined by

$$\mathbf{T}(a + bx) = (a, a - b) \qquad \text{and} \qquad \mathbf{U}(a, b) = b + ax.$$

(a) Show that $\mathbf{T}$ is invertible and determine $\mathbf{T}^{-1}$.

(b) Use any matrix representations for $\mathbf{T}$ and $\mathbf{U}$ to find a matrix representation for $\mathbf{UT}$.

4. (**7+6 pts.**) Let $\beta = \{\,1 - 2x, 1 - x\,\}$ and $\gamma = \{\,-1, 1 - x\,\}$ be two ordered bases for $\mathbb{P}_1(\mathbb{R})$, and let $\mathbf{T}$ be a linear operator on $\mathbb{P}_1(\mathbb{R})$ given by $\mathbf{T}(a + bx) = b + ax$.

(a) Find the change of coordinate matrix Q, that changes γ-coordinates into β-coordinates, and use it to evaluate $[1 + 3x]_\gamma$.

(b) Use Q to find $[\mathbf{T}]_\gamma$.