Kuwait University
First Midterm
Math 261 - Abstract Algebra

Mathematics Department
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Give full reasons for your answer. State clearly any Theorem you use.

1. (3 pts.) Let $*$ be defined on $\mathbb{Z}$ by $m * n = m^2 + n^2$ for all $m, n \in \mathbb{Z}$. Is $*$ a binary operation on $\mathbb{Z}$? Is it associative? Is it commutative? Explain you answers.

2. (3 pts.) Let $M(S)$ denote the set of all mappings from S to S for any nonempty set S. Show that the composition is an associative operation on the set of all invertible mappings in $M(S)$, with identity I_S.

3. (4 pts.) Let G be a group with operation $*$.

(a) Show that the inverse of each element in G is unique.

(b) If $a \in G$ with $a * b = b$ for some $b \in G$, then a is the identity element of G.

4. (5 pts.) Let G be a permutation group on a nonempty set S and $T \subseteq S$.

(a) Show that G_T, the element-wise stabilizer, is a subgroup of G.

(b) Assume that $S = \{1, 2, 3, 4\}$, $G = Sym(S) = S_4$ and $T = \{1, 2\}$. Compute G_T, the element-wise stabilizer, and $G_{(T)}$, the set-wise stabilizer.

5. (5 pts.) Let G be a group with operation $*$.

(a) Show that if H and K are two subgroups of G, then $H \cap K$ is a subgroup of G.

(b) Give an example to show that $H \cup K$ is not necessarily a subgroup of G.

Bonus Question (1pt):

- Write $(3\ 4\ 5)\ (1\ 2\ 5\ 4)\ (1\ 3\ 5)$ as a single cycle or a product of disjoint cycles.