Kuwait University
Second Midterm
Math 261 - Abstract Algebra

Mathematics Department
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Calculators and communication devices are NOT ALLOWED. Give full reasons for your answer.

1. (3 pts.) Let G be a permutation group on a nonempty set S, and define a relation $\sim$ on S by $a \sim b$ if and only if there exists $\alpha \in G$ with $\alpha(a) = b$. Show that $\sim$ is an equivalence relation on S.

2. (3 pts.) Show that if a is an odd integer, then $a^2 \equiv 1 \pmod 8$.

3. (4 pts.) Let G be a cyclic group.
   (a) Show that G is abelian.
   (b) Show that if $a, b \in G$ with $o(a) = m$ and $o(b) = n$, then $o(ab) = mn$.

4. (4 pts.)
   (a) Compute the distinct left cosets of $H = \langle \big((1 \;\; 2 \;\; 3), 1\big) \rangle$ in $S_3 \times \mathbb{Z}_2$.
   (b) Find the inverse of $\big((1 \;\; 2 \;\; 3), 1\big)$ in $S_3 \times \mathbb{Z}_2$.

5. (6 pts.) Let $\mathbb{U}_n = \big\{[k] \,:\, 1 \leq k < n \text{ and } GCD(k, n) = 1\big\}$.
   (a) Show that $\mathbb{U}_n$ is closed under the multiplication operation.
   (b) Find the inverse of $[7]$ as the least nonnegative integer in $\mathbb{U}_{78}$.
   (c) What is $\phi(78)$? Explain.

Bonus Question (1pt):

- Show that if $a \equiv b \pmod n$ and $c \equiv d \pmod n$, then $ac \equiv bd \pmod n$.