
Calculators and communication devices are NOT ALLOWED. Give full reasons for your answer.

1. (2 pts.) Solve the equation $(2 \ 4 \ 5) x (1 \ 4 \ 3) = (1)$ in S_5 .
2. (2 pts.) Determine the right cosets of $H = \langle (1 \ 2 \ 3) \rangle$ in S_3 .
3. (2 pts.) Let G be a group and $a \in G$ with $o(a) = n$. Show that $o(a^{-1}) = n$ as well.
4. (2 pts.) Is (\mathbb{Z}_4^*, \odot) a group? Explain.
5. (3 pts.) Let G be a group with no subgroups other than $\{e\}$ and G . Show that G is cyclic.
6. (3 pts.) Show that if a is an odd integer, then $a^2 \equiv 1 \pmod{8}$.
7. (3+1+2 pts.)
 - (a) Find the greatest common divisor of 23 and 50, denoted by $\text{GCD}(23, 50)$, and write it as a linear combination of 23 and 50.
 - (b) What is the inverse of $[23]$ in \mathbb{U}_{50} ?
 - (c) What is the order of \mathbb{U}_{50} ? Explain.

Bonus Question (1pt):

- Show that if $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $a + c \equiv b + d \pmod{n}$.