1. (3+3+2 pts.)

(a) Let A, B and C be three nonempty sets. Show that if $A \subseteq B$, then $A - C \subseteq B - C$.

(b) Show that there is no odd integer that can be expressed in the form $4m - 1$ and in the form $4n + 1$ for integers m and n.

(c) Let $\mathcal{R}$ be the equivalence relation on $\mathbb{Q}$ given by $\{ (x, y) \in \mathbb{Q} \times \mathbb{Q} : x - y \in \mathbb{Z} \}$. Find $\overline{\frac{1}{5}}$, the equivalence class of $\dfrac{1}{5}$.

2. (4 pts. each)

(a) Let $\mathcal{R}$ be some relation on a set A, and let $\mathcal{S}$ be a transitive relation containing $\mathcal{R}$. Show that $\mathcal{R} \circ \mathcal{R} \subseteq S$.

(b) Let $f : A \to B$ and $g : B \to C$ be two functions for some nonempty sets A, B, and C. Without assuming that $g \circ f$ is a function, show that if $(a, c_1), (a, c_2) \in g \circ f$, then $c_1 = c_2$, where $a \in A$, $c_1, c_2 \in C$.

3. (**4 pts. each**)

(a) Let $a_1 = 1$, $a_2 = 1$, and $a_{n+2} = a_{n+1} + a_n$ for all $n \in \mathbb{N}$. Show that a_{3n} is an even number for all natural number n.

(b) Let $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be a function given by $f(a, b) = a + b$ for all $a, b \in \mathbb{N}$. Decide whether f is one-to-one and onto $\mathbb{N}$.

4. **(4+2 pts.)** Let A and B be two disjoint denumerable sets. Assume that $f : \mathbb{N} \to A$ and $g : \mathbb{N} \to B$ are two bijections. Define $h : \mathbb{N} \to A \cup B$ by

$$h(x) = \begin{cases} f\left(\frac{x+1}{2}\right) & \text{if } x \text{ is odd,} \\ g\left(\frac{x}{2}\right) & \text{if } x \text{ is even.} \end{cases}$$

(a) Show that h is bijection.

(b) Show that $A \cup B$ is countable.

5. (**4+4+2 pts.**)

(a) Show that $A = \left\{ \frac{1}{3x+1} \ : \ x \in \mathbb{N} \right\}$ is countable.

(b) Show that $B = (3, 4) \subseteq \mathbb{R}$ is uncountable.

(c) What is the cardinality of A and B?