Kuwait University

Faculty of Science

Department of Mathematics

Math 250
Foundations of Mathematics
Summer 2022/2023

First Exam
June 15, 2023

Name	
ID Number	

__Duration__ **60** minutes (This exam contains **4** questions).

Section No.	Instructor Name
1	**Dr. Abdullah Alazemi**

Give full reasons for your answer and State clearly any Theorem you use.

Question 1	
Question 2	
Question 3	
Question 4	
Total	40

1. (10 pts.) Let **P** and **Q** be any two propositions.

    (a) Use a truth table to show that "$(\mathbf{P} \Rightarrow \mathbf{Q}) \Leftrightarrow ((\sim \mathbf{P}) \vee \mathbf{Q})$" is a tautology.

    (b) Use part (a) to find a propositional form equivalent to the negation of "$\mathbf{P} \Rightarrow \mathbf{Q}$".

2. (10 pts.) Find a denial for "$(\exists! x)[\mathbf{P}(x)]$".

3. (10 pts.)

    (a) Let $a, b, c \in \mathbb{Z}$. Show that if $a \mid b$ and $a \mid c$, then $a \mid (b - c)$.

    (b) Let W be the set of all prime numbers in $\mathbb{N}$. Show that W is infinite.

4. (10 pts.) Let $A = \{\, 4^n : n \in \mathbb{Z} \,\}$ and $B = \{\, 2^n : n \in \mathbb{Z} \,\}$. Show that A is a proper subset of B.