جامعة الكويت
KUWAIT UNIVERSITY

Kuwait University

Faculty of Science

Department of Mathematics

Math 250
Foundations of Mathematics
Summer 2022/2023

Second Exam

Monday, June 26, 2023

Name										
ID Number										

Duration 60 minutes (This exam contains **4** questions).

Section No.	Instructor Name
1	**Dr. Abdullah Alazemi**

Give full reasons for your answer and State clearly any Theorem you use.

Question 1	
Question 2	
Question 3	
Question 4	
Total	/40

1. (**10 pts.**) Let A, B and C be any three nonempty sets. Prove or disprove the following statements:

(a) $A \cap \widetilde{B} = A - B$.

(b) For any two sets A and B, if $A \times B = \phi$, then A or B is the emptyset.

2. (**10 pts.**)

(a) Show that $\widetilde{A \cap B} = \widetilde{A} \cup \widetilde{B}$.

(b) Let $A_i = \mathbb{N} - \{i+1, i+2, i+3\}$ for all $i \in \mathbb{N}$. Find: $\bigcup_{i \in \mathbb{N}} \widetilde{A_i}$ and $\bigcap_{i \in \mathbb{N}} \widetilde{A_i}$.

3. (**10 pts.**) Use a proof by induction in what follows.

(a) Show that 6 divides $n^3 - n$, for all $n \in \mathbb{N}$.

(b) Show that for all natural numbers $n > 4$, $n^2 - n - 20 \geq 0$.

4. (**10 pts.**) Let $\mathcal{R} = \{ (x, y) \in \mathbb{R} \times \mathbb{R} : y = 2x + 1 \}$ and let $\mathcal{S} = \{ (x, y) \in \mathbb{R} \times \mathbb{R} : y = x^2 - 1 \}$.

(a) Find $\mathcal{R}^{-1}$ and $\left(\mathcal{R}^{-1} \right)^{-1}$.

(b) Find $\mathcal{S} \circ \mathcal{R}$.