Kuwait University
Faculty of Science
Department of Mathematics

Math 261
Abstract Algebra I
Fall 2021/2022

Final Exam
Monday, Jan 24, 2022

Name										
ID Number										

Duration **2** hours [11:00 am - 01:00 pm] (This exam contains **5** questions).

Section No.	Instructor Name
1	**Dr. Abdullah Alazemi**

Give full justification for your answers and state clearly any theorems you use.
Calculators and communication devices are not allowed in the examination room.

Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Total	/40

1. (**3+3+2 pts.**) Let $GL_n(\mathbb{R}) = \{\,\text{all } n \times n \text{ nonsingular matrices with real entries}\,\}$ be a group with the operation of matrix multiplication.

(a) Show that $SL_n(\mathbb{R}) = \{\, A \in GL_n(\mathbb{R}) : |\,A\,| = 1 \,\}$ is a subgroup of $GL_n(\mathbb{R})$.

(b) Show that $\theta : GL_n(\mathbb{R}) \to \mathbb{R}^*$ defined by $\theta(A) = |\,A\,|$ for each $A \in GL_n(\mathbb{R})$ is a homomorphism onto $\mathbb{R}^*$.

(c) Use the Fundamental Homomorphism Theorem to show that $GL_n(\mathbb{R})/SL_n(\mathbb{R})$ is isomorphic to $\mathbb{R}^*$.

2. (**3+3+2 pts.**)

(a) Find the centralizer of $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \in GL_2(\mathbb{R})$, denoted as $C\,(X)$.

(b) Let φ denote the Euler phi-function. Compute $\varphi(21)$.

(c) Simplify $\left([1], (1\ \ 2\ \ 3)^{-1} \right)^{-1} (\,[1], e\,)(\,[0], (1\ \ 3\ \ 2)\,)$ in $\mathbb{Z}_4 \times S_4$.

3. (**4 pts. each**)

(a) Let G and H be two groups with a homomorphism $\theta : G \to H$. Show that the subgroup $\ker\theta$ is normal in G.

(b) Let G be a group with a normal subgroup N. Show that the mapping $\theta : G \to G/N$ defined by $\theta(a) = aN$ for each $a \in G$ is a homomorphism of G onto G/N, and $\ker\theta = N$.

4. (**4 pts. each**) Let a be a fixed element in a group G. Define $\gamma_a : G \to G$ by $\gamma_a(x) = a\,x\,a^{-1}$. Then

(a) Show that γ_a is an isomorphism.

(b) Show that if $a, b \in G$, then $\gamma_a\gamma_b = \gamma_{ab}$.

5. (**4 pts. each**) Let G be a nonabelian group and let $Z(G)$ denote the center of G.

(a) Show that the factor group $G/Z(G)$ is not cyclic.

(b) Show that if $|\,G\,| = pq$ where p and q are primes, then $Z(G)$ is trivial.

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