Kuwait University
Faculty of Science
Department of Mathematics

Math 261
Abstract Algebra I
Fall 2021/2022

First Exam
Wednesday, Dec 1, 2021

Name										
ID Number										

Duration 90 minutes (This exam contains **5** questions).

Section No.	Instructor Name
1	**Dr. Abdullah Alazemi**

Give full justification for your answers and state clearly any theorems you use. Calculators and communication devices are not allowed in the examination room.

Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Total	50

1. (**10 pts.**) Let S_n denote the symmetric group on $\{1, 2, \ldots, n\}$ for positive integer n.

(a) Compute $(1\ \ 5)(2\ \ 5\ \ 3)(4\ \ 6)(2\ \ 3\ \ 5)(4\ \ 6)$.

(b) Find x in

$$(1\ \ 2\ \ 3)\, x\, (4\ \ 6\ \ 5) = (4\ \ 5\ \ 6).$$

(c) Show that S_n is non-abelian for $n \geq 3$.

2. (**10 pts.**) Let G be a group with the identity e.

(a) Show that the inverse of each element in G is unique.

(b) Show that if $x^2 = e$ for every $x \in G$, then G must be abelian.

3. (**10 pts.**) Let G be a permutation group on a set S and let $T \subseteq S$.

(a) Show that $G_{(T)}$, the setwise stabilizer of T, is a subgroup of G.

(b) If $S = \{1, 2, \ldots, 10\}$ and $T = \{2, 3, 5\}$, find $G_{(T)}$ and its order.

4. (**12 pts.**)

(a) Show that if x is an odd integer, then $x^2 \equiv 1 \pmod 8$.

(b) Let H be a subgroup of a group G. For $a, b \in G$, let $a \sim b$ if and only if $ab^{-1} \in H$.
Show that $\sim$ is an equivalence relation on G.

5. (**8 pts.**) Let $\mathcal{B}(X)$ denote the family of all subsets of a nonempty set X. For any $A, B \in \mathcal{B}(X)$, define the operation $\triangle$ by

$$A \triangle B = (A - B) \cup (B - A).$$

Assuming that $\triangle$ is an associative operation on $\mathcal{B}(X)$, show that $\mathcal{B}(X)$ is an abelian group with the operation $\triangle$.