Kuwait University
Faculty of Science
Department of Mathematics

Math 261
Abstract Algebra I
Fall 2021/2022

Second Exam
Sunday, Jan 09, 2022

Name										
ID Number										

Duration 90 minutes (This exam contains **5** questions).

Section No.	Instructor Name
1	**Dr. Abdullah Alazemi**

Give full justification for your answers and state clearly any theorems you use.
Calculators and communication devices are not allowed in the examination room.

Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Total	50

1. (**10 pts.**) Let $\mathbb{U}_n = \{ [k] : 1 \leq k < n \text{ and } gcd(k,n) = 1 \}$.

(a) Use the Euclidean algorithm to find the inverse of [21] as the least nonnegative integer in $\mathbb{U}_{100}$.

(b) Find the order of $\mathbb{U}_{100}$.

2. (**10 pts.**) Let G be a group.

(a) Show that if G is cyclic, then G is abelian.

(b) Show that if $a \in G$ is a generator for G, then a^{-1} is a generator for G as well.

3. (**10 pts.**) Let H be a subgroup of a group G and let $a \in G$. Then

(a) Show that $aH = H$ if and only if $a \in H$.

(b) If $H = \langle (1\ \ 2\ \ 3) \rangle \times \langle [1] \rangle$ and $G = S_3 \times \mathbb{Z}_2$, compute all left cosets of H in G and find its index.

4. (**10 pts.**) Let G be a group.

(a) Show that if $H \leq G$ with index 2, then $aHa^{-1} = H$ for any $a \in G$.

(b) Use Fermat's Little Theorem to find the least nonnegative integer x so that $5^{2022} \equiv x \pmod{11}$.

(c) Suppose that G is a non-abelian group of order 10. Show that G has an element of order 5.

5. (**10 pts.**)

(a) Let $\theta : G \to H$ be a group homomorphism mapping. Show that $\theta(e_G) = e_H$ and $\theta(a^{-1}) = \theta(a)^{-1}$ for each $a \in G$.

(b) Show that the group of integers $\mathbb{Z}$ is isomorphic to the multiplicative group $M = \{ 2^m : m \in \mathbb{Z} \}$.